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ORIGINAL RESEARCH

Stability, Tracking Control, and Synchronisation of Hyperchaotic 5D Lorenz System using Active Backstepping Designs

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Abstract

Introduction: Nonlinear dynamical systems show complex behaviour sensitive to initial conditions. Controlling and synchronising chaos has applications in engineering, science, secure communications, encryption, biology, chemistry, finance, neural networks, cryptography, and medicine.

Aim: This paper aims to analyze the synchronisation, tracking control, and stability of hyperchaotic 5D Lorenz systems. A linear feedback controller is built to ensure asymptotic stability of the two identical hyperchaotic 5D Lorenz systems developing from distinct initial conditions, based on Lyapunov stability theory and active backstepping nonlinear approaches.

Methods: The three positive Lyapunov exponents and complex dynamical behaviour of the hyperchaotic 5D system are demonstrated. The control functions for the corresponding control and synchronisation of the hyperchaotic 5D Lorenz system are designed using the active backstepping nonlinear technique. The nonlinear controllers of the intended backstepping can stabilise and direct the hyperchaotic 5D Lorenz system at any place to follow any smooth function of time trajectory. The proposed method integrates the selection of a Lyapunov function with the creation of active control, and it is a systematic design technique. To validate the feasibility and effectiveness of the proposed control technique, numerical simulation results are presented.

Conclusion: The use of an active backstepping control approach to control and synchronise the hyperchaotic 5D Lorenz system stabilises chaotic motion, simplifies design without needing eigenvalues, and efficiently controls high-dimensional hyperchaotic systems, outperforming conventional chaos. Thus, numerical simulations confirm effectiveness.

Keywords: Hyperchaotic system; Lyapunov exponents; tracking control; synchronization; active backstepping;

All co-authors agreed to have their names listed as authors.

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1. INTRODUCTION

In recent years, it has been established that nonlinear dynamical systems exhibit complex irregular behaviour in real life whenever their evolution is sensitive to the initial conditions [1]. Even with minuscule parameter changes, these complex systems can produce dramatically divergent chaotic paths [2]. In contrast to regular chaotic systems, hyperchaotic systems exhibit more complex dynamic behaviour and possess extensive application values due to their distinctive characteristic of having two or more positive Lyapunov exponents [3]. Consequently, control and synchronization of chaotic systems have a wide range of uses in engineering and science, including secure communications [4] , optical communication [5], voice and image encryption [6, 7], biology [8], chemistry [9], finance [10], neural networks [11, 12], cryptography [13, 14], lasers [15], and medicine [16].

In the aspect of control and synchronization, various control approaches such as active control [17-21], sliding mode control [22-24], digital redesign control [25-27], optimal control [28], backstepping method [29-35], impulsive control [36], intermittent scheme [37-40], event triggered [41-43], switching process [44], composite nonlinear feedback [45], finite-time [46, 47] and neural-based control [48, 49] have been proposed to satisfy stability, robustness, and suitable performances of the systems.

Specifically, backstepping nonlinear control technique stands out among control/synchronization schemes with linear and nonlinear control inputs that have been developed over time for its capacity to achieve global stability, tracking, and transient performance for a wide class of strict-feedback nonlinear systems. The backstepping method has many benefits, including being applicable to a wide range of chaotic systems whether or not they contain external excitation; requiring only one controller to achieve synchronization between coupled chaotic systems because it reduces controller complexity; the controller is singularity free from the nonlinear term of quadratic type; flexibility to construct a control law that can be extended to higher dimensional hyperchaotic systems; and the closed-loop nature of the backstepping method. But this method can only be used with rigorous feedback mechanisms [50]. In order to apply the backstepping technique to no strict feedback systems, the active backstepping nonlinear technique based on Lyapunov stability theory was developed to synchronize the hyperchaotic systems [50]. Due to its intriguing possibilities, including guaranteed stability, computational simplicity, and robustness against uncertainties, active backstepping has surprisingly attracted significant attention in a variety of applications [51].

Using two identical hyperchaotic systems, we design an active backstepping control in this paper that can guarantee global stabilities, tracking, and global synchronization. The stability of the synchronized state in the higher order hyperchaotic system is examined using the active backstepping approach and the Lyapunov stability theorem. We create an appropriate controller and use active control to accomplish drive-response synchronization in order to get around mathematical challenges.

The paper is organized as follows. The next section presents the physical system and the corresponding mathematical model. Further characterization of the system is discussed. The control and synchronization scheme is presented in section 3, section 4 is devoted to numerical results and discussions and the paper is concluded in section 5.

2. MATERIAL AND METHODS

2.1 MATHEMATICAL MODEL AND CHARACTERIZATION

Here, we consider the following simple 5D quadratic smooth autonomous system:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 + x_5 \\ \dot{x}_2 &= bx_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \\ \dot{x}_4 &= dx_4 - x_1x_5 \\ \dot{x}_5 &= qx_1 \end{aligned} \tag{1}$$

where $x_{i=1,2,3,4,5}$ are the states and a, b, c, d, q are real coefficients, where dots indicate a derivative of time t. The 5D system (1) is hyperchaotic for the following parameter values $a = 10;\ b = 28;\ c = \frac{8}{3};\ d = 1.3;\ q = 2.5$.

According to the reference [52], in order for a hyperchaotic system to exist, a few requirements must be met: (a) the system must be dissipative; (b) its dimension must be at least 4; and (c) it must contain at least two equations that contribute to the instability, each of which must have at least one nonlinear term.

Here, $L_1 = 0.391380; L_2 = 0.243213;\ L_3 = 0.033992;\ L_4 = -0.000110;\ L_5 = -13.032701$ are the Lyapunov exponents. The system's Lyapunov exponent now has two negative and three positive exponents of the Lyapunov characteristic. Evidently, the sum of all Lyapunov exponents is smaller than zero $(\text{i. e.} L = -12.364226)$. Consequently, the system has reached a hyperchaotic state.

3 THEORETICAL ANALYSIS OF THE PROPOSED HYPERCHAOTIC SYSTEM

The equilibrium and stability analysis of the hyperchaotic system is first presented, and as a result, the suggested system is examined by the Lyapunov exponent spectrum. This results in an analysis of the dynamic behaviour of the hyperchaotic system.

3.1 Equilibria and Stability

Identifying the system's equilibrium points is a useful place to start when analyzing it. From there, use the related linearized systems to describe the local dynamic behaviour of the system orbits close to these points. The system's nonlinear dynamics are significantly influenced by the equilibria's local (linearized) dynamical properties and spatial distribution. The following algebraic equations can be solved simultaneously to determine the equilibria of system (1):

$$\begin{aligned} a(x_2 - x_1) + x_4 + x_5 &= 0 \\ bx_1 - x_2 - x_1x_3 &= 0 \\ -cx_3 + x_1x_2 &= 0 \\ dx_4 - x_1x_5 &= 0 \\ qx_1 &= 0 \end{aligned} \tag{2}$$

Solving the system (2), the equilibrium point $\hat{O}(0,0,0,0,0)$.

3.2 Control of the Hyperchaotic 5D System

3.2.1 Active Backstepping Control

In the following, the backstepping technique is employed to design an active controller for the hyperchaotic system presented by the equation (1) to the origin. According to the active control theory, the controlled hyperchaotic system can be written in the following form

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 + x_5 + u_1 \\ \dot{x}_2 &= bx_1 - x_2 - x_1x_3 + u_2 \\ \dot{x}_3 &= -cx_3 + x_1x_2 + u_3 \\ \dot{x}_4 &= dx_4 - x_1x_5 + u_4 \\ \dot{x}_5 &= qx_1 + u_5 \end{aligned} \tag{3}$$

where $u = [u_1, u_2, u_3, u_4, u_5]^T$ is the active control function. In practical applications, the controller to be designed must be simple, efficient and easy to implement. Therefore, let $u_1 = 0$ and $u_5 = 0$, then the controlled dynamics can be written as

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 + x_5 \\ \dot{x}_2 &= bx_1 - x_2 - x_1x_3 + u_2 \\ \dot{x}_3 &= -cx_3 + x_1x_2 + u_3 \\ \dot{x}_4 &= dx_4 - x_1x_5 + u_4 \\ \dot{x}_5 &= qx_1 \end{aligned} \tag{4}$$

Now our objective is to find the control inputs u_2, u_3 and u_4 that enable the state vectors $x_i = [x_1, x_2, x_3, x_4, x_5]^T$ converge to zero as time t goes to infinity. To achieve such goal, the backstepping design method is adopted.

The backstepping design procedure is recursive. At the i th-step, the i th-order subsystem is stabilized with respect to a Lyapunov function and the control input function V_i by the design of a virtual control α_i and the control input function u_i. Now, the design of the active controller based on the backstepping technique is begun as follows.

Step 1. Let $z_1 = x_1$ then we can obtain its derivative as follows

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 \\ &= a(x_2 - x_1) + x_4 + x_5 \end{aligned} \quad (5)$$

where $ax_2 + x_4 + x_5 = \alpha_1(z_1)$ is regarded as a virtual control input.

For the design of α_1 to stabilize z_1-subsystem (5), Lyapunov function V_1 is chosen as

$$V_1 = \frac{1}{2}z_1^2 \quad (6)$$

The derivative of V_1 is obtained as

$$\begin{aligned} \dot{V}_1 &= z_1\dot{z}_1 \\ \dot{V}_1 &= -az_1^2 + \alpha_1 z_1 \end{aligned} \quad (7)$$

If $\alpha_1 = 0$ is chosen, then $\dot{V}_1$ is negative definite. This implies that the z_1-subsystem (5) is asymptotically stable. Since the virtual control function α_1 is estimative, the error between x_2 and α_1 is

$$z_2 = x_2 - \alpha_1 \quad (8)$$

Then, we can obtain the following (z_1, z_2) subsystem

$$\begin{aligned} \dot{z}_1 &= -az_1 + az_2 \\ \dot{z}_2 &= (b - x_3)z_1 - z_2 + u_2 \end{aligned} \quad (9)$$

where $x_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2. In this step, stabilizing the (z_1, z_2)-subsystem (9), Lyapunov function V_2 can be chosen as follows

$$V_2 = V_1 + \frac{1}{2}z_2^2 \quad (10)$$

Its derivative is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 \\ \dot{V}_2 &= -az_1^2 - z_2^2 + z_2((a + b - x_3)z_1 + u_2) \end{aligned} \quad (11)$$

If $u_2 = (x_3 - a - b)z_1$ and $\alpha_2 = 0$, then $\dot{V}_2 = -az_1^2 - z_2^2 < 0$ makes (z_1, z_2)-subsystem (9) asymptotically stable. Similarly, assume that $z_3 = x_3 - \alpha_2$, then we can derive the following (z_1, z_2, z_3) -subsystem.

$$\begin{aligned} \dot{z}_1 &= -az_1 \\ \dot{z}_2 &= -z_2 \\ \dot{z}_3 &= -cz_3 + z_1z_2 + u_3 \end{aligned} \quad (12)$$

Step 3. In order to stabilize the (z_1, z_2, z_3)-subsystem (12), a Lyapunov function V_3 can be chosen as follows

$$V_3 = V_2 + \frac{1}{2}z_3^2 \quad (13)$$

The derivative of V_3 is

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + z_3\dot{z}_3 \\ \dot{V}_3 &= -az_1^2 - z_2^2 - cz_3^2 + z_3(z_1z_2 + u_3) \end{aligned} \quad (14)$$

If $u_3 = -z_1 z_2$ and $\alpha_3 = 0$, is chosen, then $\dot{V}_3 = -az_1^2 - z_2^2 - cz_3^2 < 0$ makes the (z_1, z_2, z_3)-subsystem (12) asymptotically stable. Similarly, assume that $z_4 = x_4 - \alpha_3$, then we can derive the following (z_1, z_2, z_3, z_4) -subsystem.

$$\begin{aligned} \dot{z}_1 &= -az_1 \\ \dot{z}_2 &= -z_2 \\ \dot{z}_3 &= -cz_3 \\ \dot{z}_4 &= dz_4 - z_1 x_5 + u_4 \end{aligned} \tag{15}$$

Step 4. In order to stabilize the (z_1, z_2, z_3, z_4) -subsystem (15), a Lyapunov function V_4 can be chosen as follows

$$V_4 = V_3 + \frac{1}{2} z_4^2 \tag{16}$$

The derivative of V_4 is given by

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + z_4 \dot{z}_4 \\ \dot{V}_4 &= -az_1^2 - z_2^2 - cz_3^2 - dz_4^2 + z_4(2dz_4 - z_1 x_5 + u_4) \end{aligned} \tag{17}$$

If $u_4 = z_1 x_5 - 2dz_4$, is chosen, then $\dot{V}_4 = -az_1^2 - z_2^2 - cz_3^2 - dz_4^2 < 0$ makes the (z_1, z_2, z_3, z_4) -subsystem (15) asymptotically stable. Since $\dot{V}_4$ is negative definite, it follows that the equilibrium (0, 0, 0, 0) of the subsystem (15) is global asymptotically stable. Furthermore, since $z_1 = x_1$, $z_2 = x_2 - \alpha_1 = x_2$, $z_3 = x_3 - \alpha_2 = x_3$ and $z_4 = x_4 - \alpha_3 = x_4$. Thus, x_1, x_2, x_3 and x_4 go to zeros asymptotically as well. According to $x_1 \to 0,\ x_2 \to 0,\ x_3 \to 0,\ x_4 \to 0$ and the fifth equation of system (4), it is obtained that $(x_1, x_2, x_3, x_4, x_5)$ in the controlled system (4) tend to (0, 0, 0, 0, 0) as $t \to \infty$. In other words, the controlled system (4) is asymptotically stable with the proposed control inputs.

3.3 Synchronization of Hyperchaotic 5D System

3.3.1. Active Backstepping Synchronization

In this section, the backstepping method is utilized to design an active controller to synchronize two identical Lorentz hyperchaotic systems. In order to observe the synchronization behaviour in the Lorentz hyperchaotic system, the drive system is assumed as

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 + x_5 \\ \dot{x}_2 &= bx_1 - x_2 - x_1 x_3 \\ \dot{x}_3 &= -cx_3 + x_1 x_2 \\ \dot{x}_4 &= dx_4 - x_1 x_5 \\ \dot{x}_5 &= qx_1 \end{aligned} \tag{18}$$

and the response system

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_4 + y_5 + u_1 \\ \dot{y}_2 &= by_1 - y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -cy_3 + y_1 y_2 + u_3 \\ \dot{y}_4 &= dy_4 - y_1 y_5 + u_4 \\ \dot{y}_5 &= qy_1 + u_5 \end{aligned} \tag{19}$$

where $u = [u_1, u_2, u_3, u_4, u_5]^T$ is the active control function. In practical applications, the controller to be designed should be simple, efficient and easy to implement. Therefore, let $u_1 = 0$ and $u_5 = 0$, then the controlled dynamics can be written as

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_4 + y_5\\ \dot{y}_2 &= by_1 - y_2 - y_1 y_3 + u_2\\ \dot{y}_3 &= -cy_3 + y_1 y_2 + u_3\\ \dot{y}_4 &= dy_4 - y_1 y_5 + u_4\\ \dot{y}_5 &= qy_1\end{aligned} \tag{20}$$

Here, determining the controllers u_2, u_3 and u_4 which are required for the controlled response system (20) to synchronize with the drive system (18) is aimed at. For this purpose, let the error states between the state variables of the response system (20) and the drive system (18) be

$$e_{i=(1,2,3,4,5)} = y_i - x_i \tag{21}$$

Subtracting (18) from (20), the following error dynamics are obtained

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + e_4 + e_5\\ \dot{e}_2 &= be_1 - e_2 - e_1 e_3 - x_1 e_3 - x_3 e_1 + u_2\\ \dot{e}_3 &= -ce_3 + e_1 e_2 + x_1 e_2 + x_2 e_1 + u_3\\ \dot{e}_4 &= de_4 - e_1 e_5 - x_1 e_5 - x_5 e_1 + u_4\\ \dot{e}_5 &= qe_1\end{aligned} \tag{22}$$

The next is to find the control inputs u_2, u_3 and u_4 that make the state vectors $u_i = [u_1, u_2, u_3, u_4, u_5]^T$ converge to zero as time t goes to infinity. In order to achieve such goal, the backstepping design method is adopted.

This implies that the trajectory of the response system (20) asymptotically approaches the trajectory of the drive system (18). Furthermore, the active controller based on the backstepping method outlined in subsection 3.3.1 is designed.

Step 1. Let $z_1 = e_1$ then we can obtain its derivative as follows

$$\begin{aligned}\dot{z}_1 &= \dot{e}_1\\ &= -ae_1 + ae_2 + e_4 + e_5\end{aligned} \tag{23}$$

where $e_2 = \alpha_1(z_1)$ is regarded as a virtual control input.

For the design of α_1 to stabilize z_1-subsystem (23), Lyapunov function V_1 is chosen as

$$V_1 = \frac{1}{2} z_1^2 \tag{24}$$

The derivative of V_1 is obtained as

$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1\\ \dot{V}_1 &= -az_1^2 + (e_4 + e_5 + a\alpha_1) z_1\end{aligned} \tag{25}$$

If $\alpha_1 = 0$ is chosen, then $\dot{V}_1 = -az_1^2 + z_1(e_4 + e_5)$. The second term $(e_4 + e_5)$ in $\dot{V}_1$ will be eliminated at the next step. This implies that the z_1-subsystem (23) is asymptotically stable. Since the virtual control function α_1 is estimative, the error between e_2 and α_1 is given by

$$z_2 = e_2 - \alpha_1 \tag{26}$$

Then, we can obtain the following (z_1, z_2) subsystem

$$\begin{aligned}\dot{z}_1 &= -az_1 + az_2\\ \dot{z}_2 &= (b - e_3 - x_3) z_1 - z_2 - x_1 e_3 + u_2\end{aligned} \tag{27}$$

where $e_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2. In this step, stabilizing the (z_1, z_2)-subsystem (27). Lyapunov function V_2 can be chosen as follows

$$V_2 = V_1 + \frac{1}{2}z_2^2 \quad (28)$$

The derivative is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 \\ \dot{V}_2 &= -az_1^2 - z_2^2 + z_2((a+b-x_3)z_1 + u_2) \end{aligned} \quad (29)$$

If $u_2 = (x_3 - b - a)z_1$ and $\alpha_2 = 0$, then $\dot{V}_2 = -az_1^2 - z_2^2 < 0$ makes (z_1, z_2)-subsystem (29) asymptotically stable.

Similarly, assume that $z_3 = e_3 - \alpha_2$, then we can derive the following (z_1, z_2, z_3) -subsystem.

$$\begin{aligned} \dot{z}_1 &= -az_1 \\ \dot{z}_2 &= -z_2 \\ \dot{z}_3 &= -cz_3 + z_1z_2 + x_2z_1 + x_1z_2 + u_3 \end{aligned} \quad (30)$$

Step 3. In order to stabilize the (z_1, z_2, z_3)-subsystem (30), a Lyapunov function V_3 can be chosen as follows

$$V_3 = V_2 + \frac{1}{2}z_3^2 \quad (31)$$

The derivative of V_3 is given by

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + z_3\dot{z}_3 \\ \dot{V}_3 &= -az_1^2 - z_2^2 - cz_3^2 + z_3(z_1z_2 + x_2z_1 + x_1z_2 + u_3) \end{aligned} \quad (32)$$

If $u_3 = -z_1z_2 - x_2z_1 - x_1z_2$ and $\alpha_3 = 0$, is chosen, then $\dot{V}_3 = -az_1^2 - z_2^2 - cz_3^2 < 0$ makes the (z_1, z_2, z_3)-subsystem (30) asymptotically stable.

Similarly, assume that $z_4 = e_4 - \alpha_3$, then we can derive the following (z_1, z_2, z_3, z_4) -subsystem.

$$\begin{aligned} \dot{z}_1 &= -az_1 \\ \dot{z}_2 &= -z_2 \\ \dot{z}_3 &= -cz_3 \\ \dot{z}_4 &= dz_4 - z_1e_5 - z_1x_5 - x_1e_5 + u_4 \end{aligned} \quad (33)$$

Step 4. In order to stabilize the (z_1, z_2, z_3, z_4) -subsystem (33), a Lyapunov function V_4 can be chosen as follows

$$V_4 = V_3 + \frac{1}{2}z_4^2 \quad (34)$$

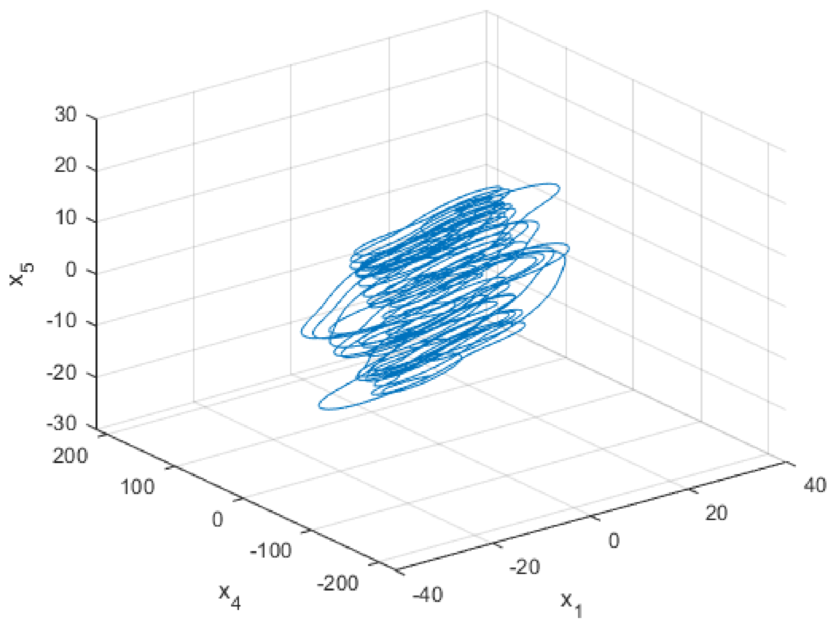
The derivative of V_4 is obtained as

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + z_4\dot{z}_4 \\ \dot{V}_4 &= -az_1^2 - z_2^2 - cz_3^2 - dz_4^2 + z_4(2dz_4 - z_1e_5 - z_1x_5 - x_1e_5 + u_4) \end{aligned} \quad (35)$$

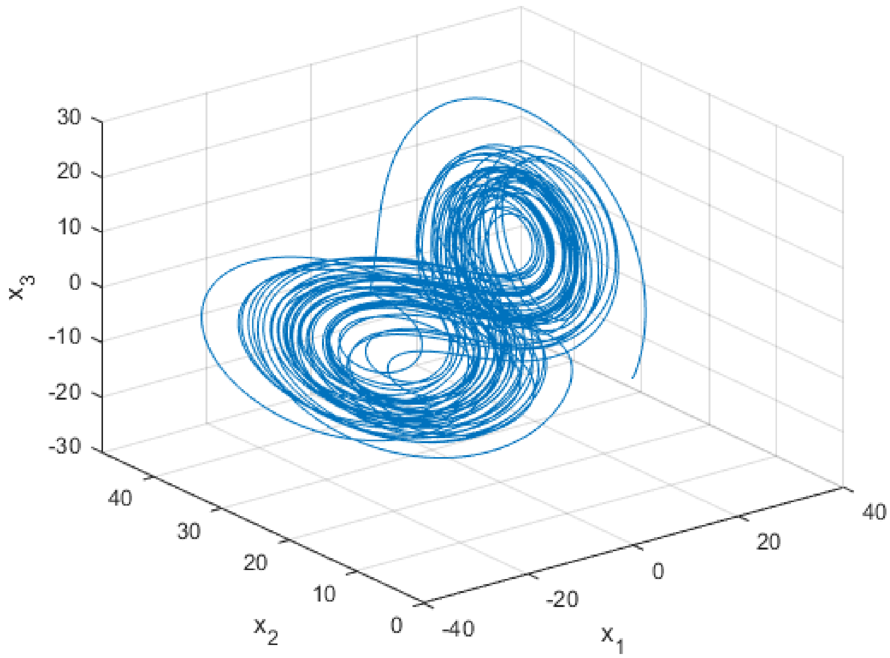
If $u_4 = z_1e_5 + z_1x_5 + x_1e_5 - 2dz_4$, is chosen, then $\dot{V}_4 = -az_1^2 - z_2^2 - cz_3^2 - dz_4^2 < 0$ makes the (z_1, z_2, z_3, z_4) -subsystem (33) asymptotically stable. Since $\dot{V}_4$ is negative definite, it follows that the equilibrium (0, 0, 0, 0) of the subsystem (33) is global asymptotically stable. Furthermore, since $z_1 = e_1,\ z_2 = e_2 - \alpha_1 = e_2,\ z_3 = e_3 - \alpha_2 = e_3$ and $z_4 = e_4 - \alpha_3 = e_4$, e_1, e_2, e_3 and e_4 go to zeros asymptotically as well. According to $e_1 \to 0,\ e_2 \to 0,\ e_3 \to 0,\ e_4 \to 0$ and the fifth equation of system (22), it is obtained that $(x_1, x_2, x_3, x_4, x_5)$ in the controlled system (22) tend to (0, 0, 0, 0, 0) as $t \to \infty$. In other words, the trajectory of the controlled response system (20) asymptotically approaches the trajectory of the drive system (18) with the proposed control inputs.

4 NUMERICAL SIMULATIONS AND ANALYSES

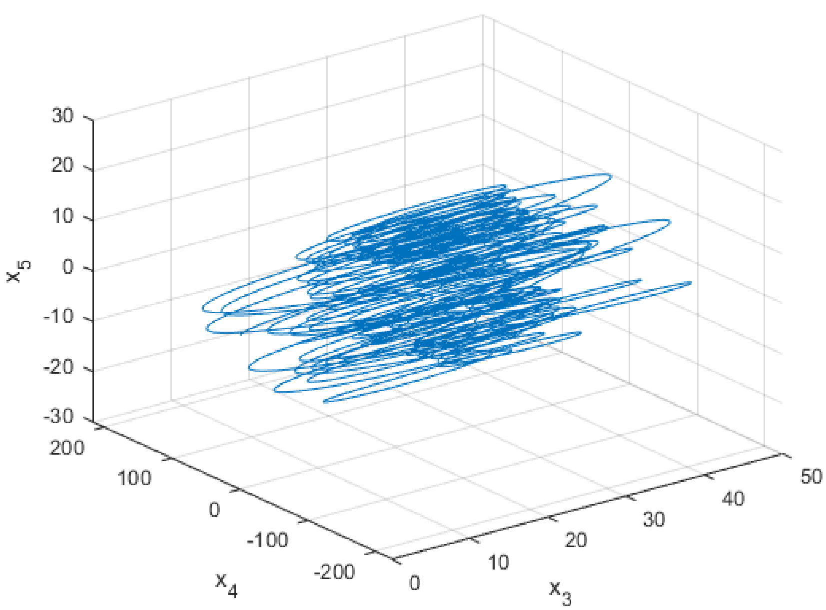
The fourth order Runge-Kutta routine is used in numerical simulation to guarantee hyperchaotic behaviour for the 5D Lorenz system with the following initial conditions:
$(x_1(0), y_1(0)) = (0.1, 1)$; $(x_2(0), y_2(0)) = (0.2, 2)$; $(x_3(0), y_3(0)) = (0.3, 3)$; $(x_4(0), y_4(0)) = (0.4, 4)$; $(x_5(0), y_5(0)) = (0.5, 4.5)$,
with a time-step of 10^{-8}, the parameter $a = 10$; $b = 28$; $c = {}^{8}/_{3}$; $d = 1.3$; $q = 2.5$ are fixed, as shown in Figures 1$(a - i)$ below. Figures 1(a - i) elucidates the intricate dynamics of the hyperchaotic 5D Lorenz system, exemplifying the pronounced sensitivity to initial conditions, a paradigmatic characteristic of chaotic behaviour. The phase portraits of the 5D Lorenz system offer a profound understanding of its underlying dynamics, revealing the existence of attractors, repellents, and other invariant sets that govern its complex behaviour.



(a) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_1, x_4, x_5)



(a) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_1, x_2, x_3)



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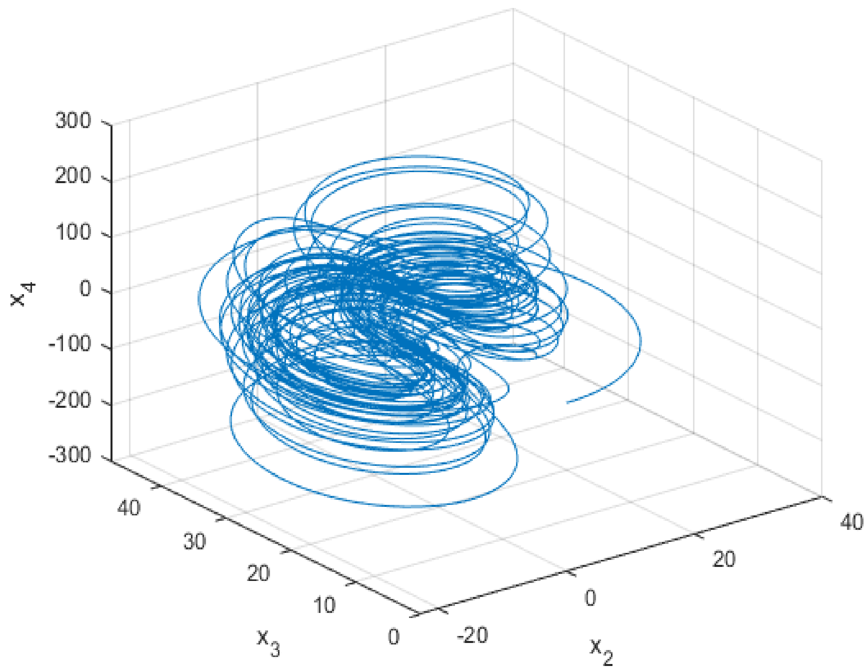
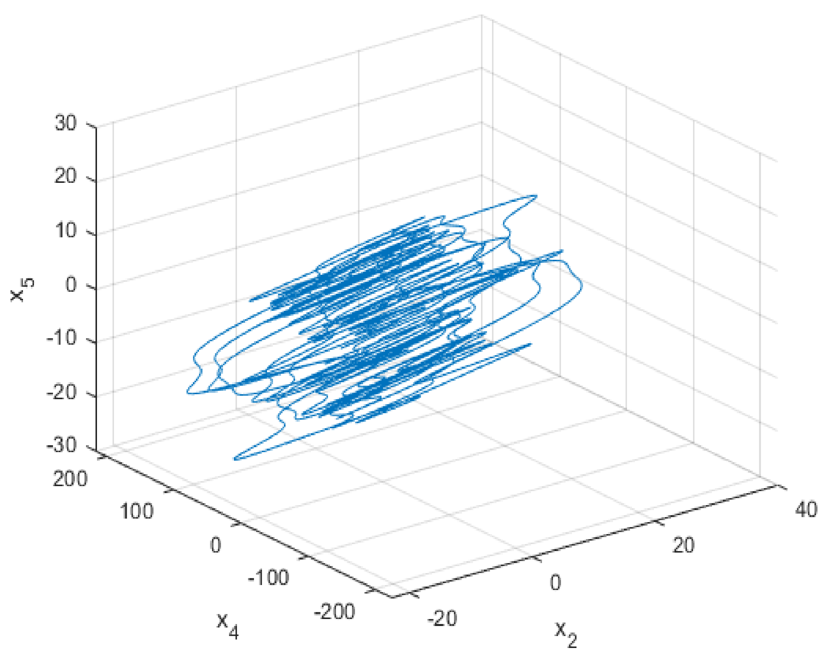
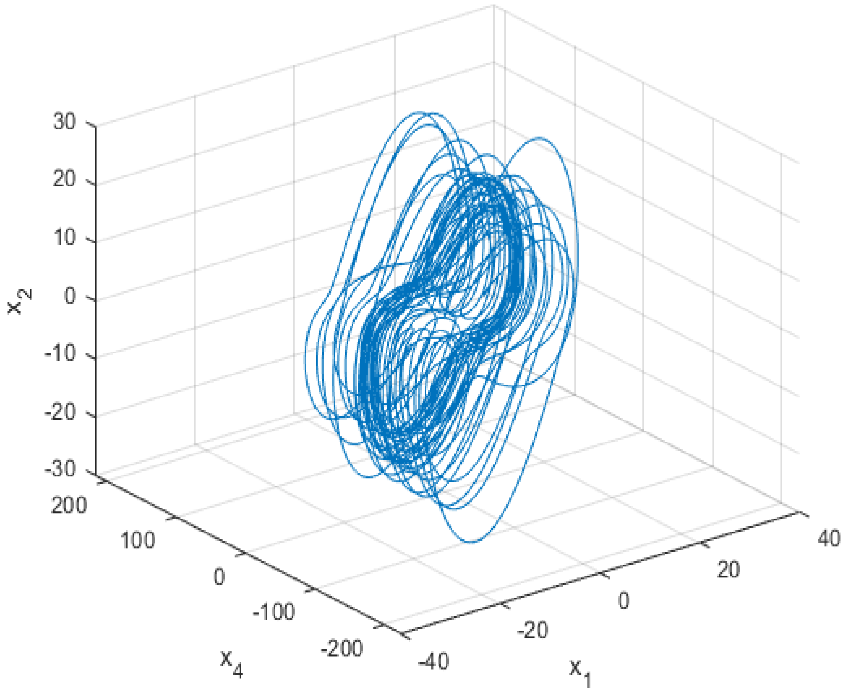
(c) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_3, x_4, x_5)



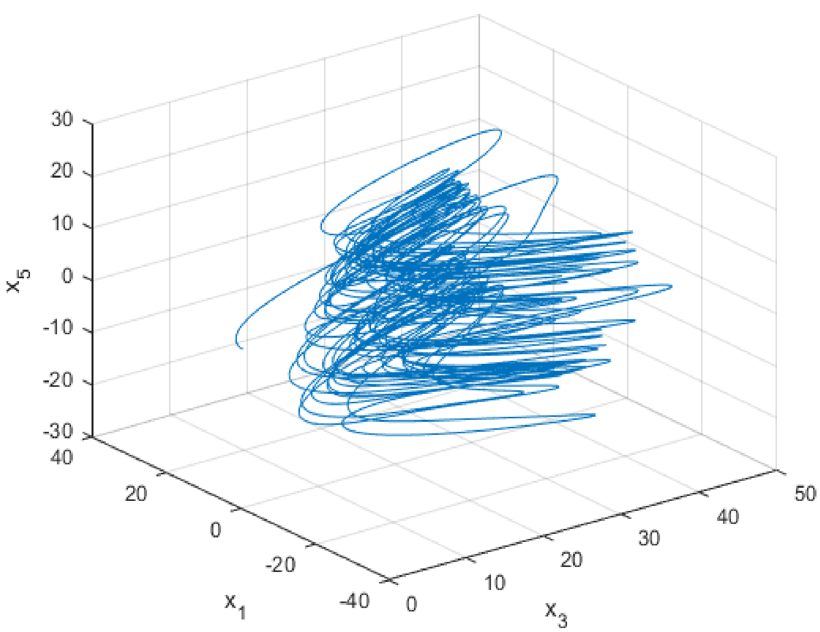
(d) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_2, x_3, x_4)



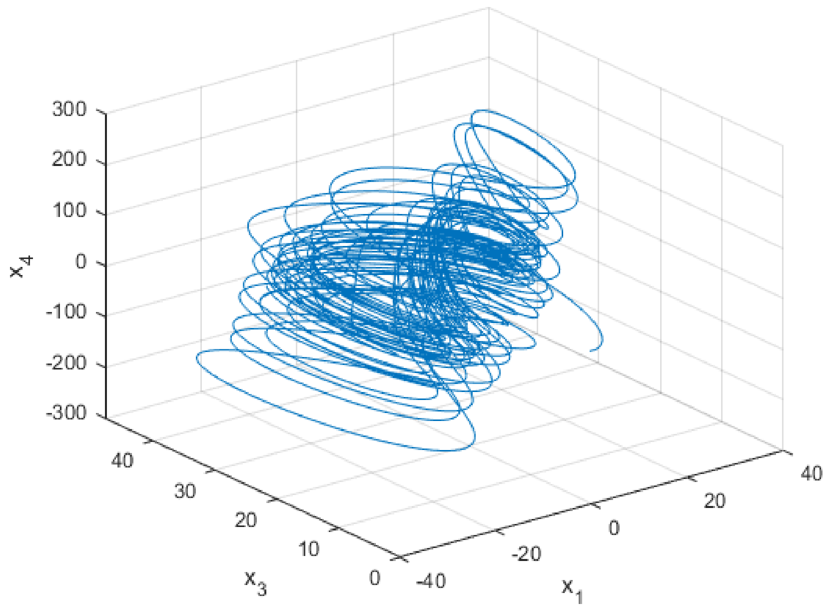
(e) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_2, x_4, x_5)



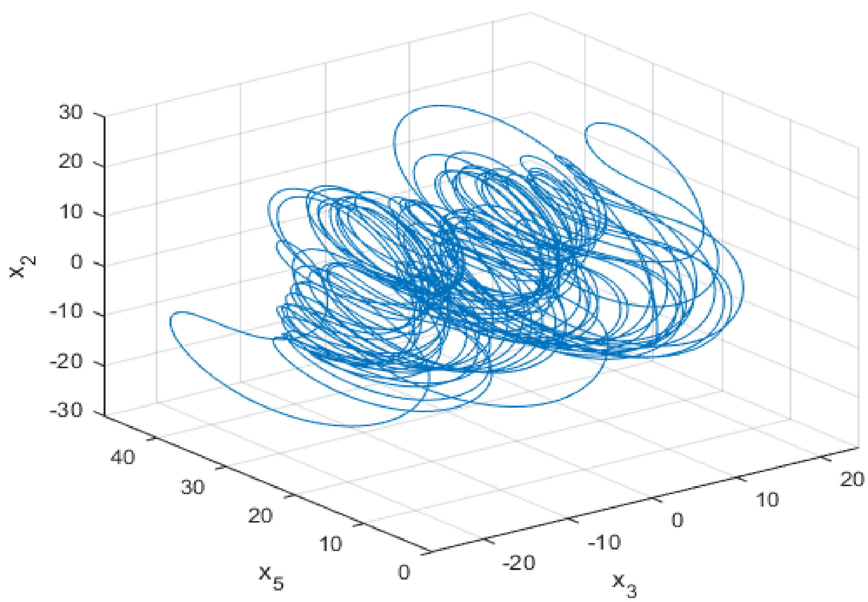
(f) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_1, x_4, x_2)



(g) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_3, x_1, x_5)



(h) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_1, x_3, x_4)



(i) 3-D Projection of the 5D hyperchaotic Lorenz system on (x_3, x_5, x_2)

Figure 1 (a – i): 3-D Phase portraits of the hyperchaotic 5D Lorenz system

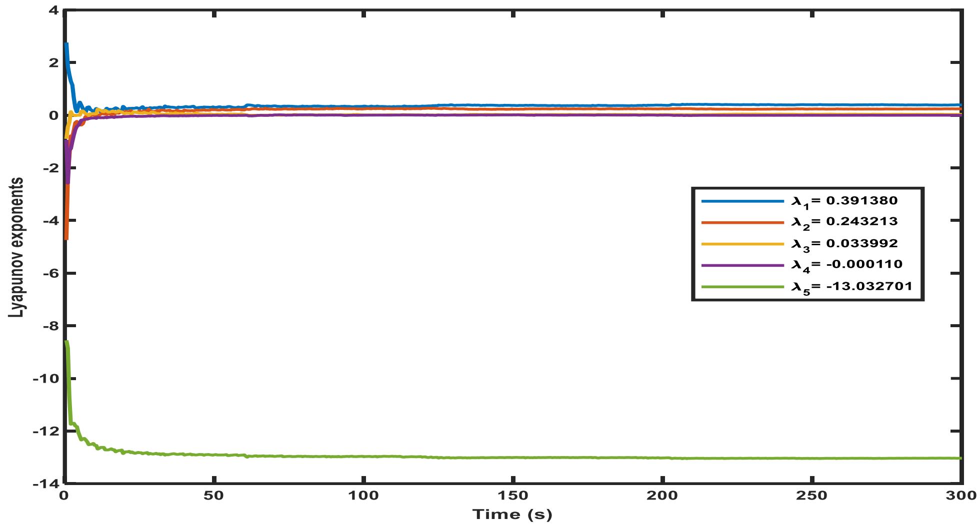


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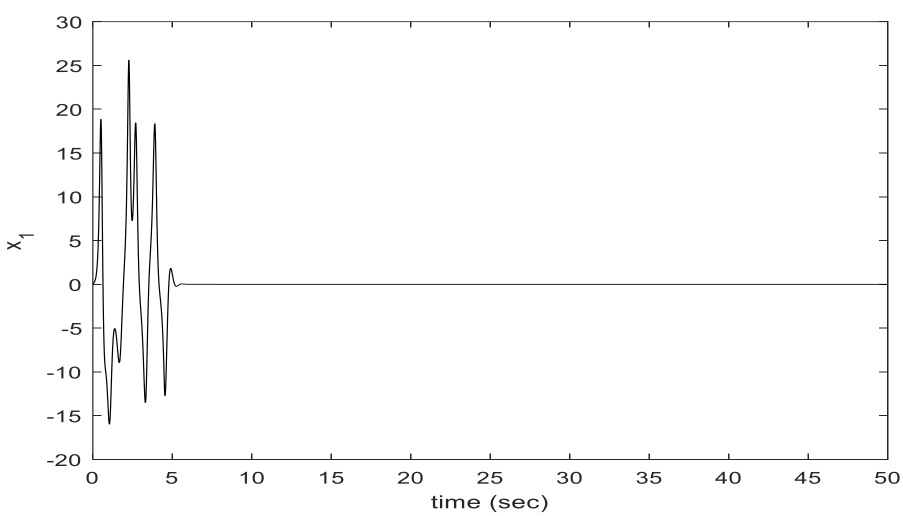
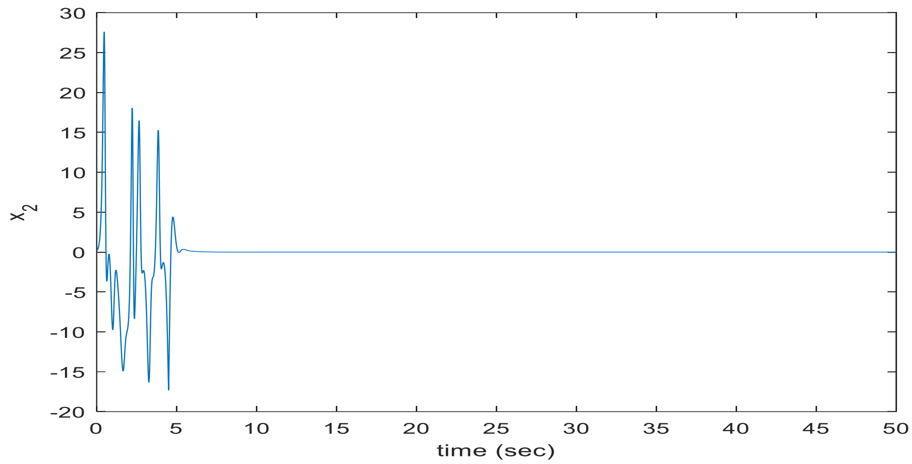
Figure 2: Dynamics of Lyapunov exponents

Figure 2 above reveals the dynamics of the Lyapunov exponents for the hyperchaotic 5D Lorenz system. Specifically, the existence of multiple positive Lyapunov exponents confirms the hyperchaotic nature of the system, characterized by divergence in multiple directions. The magnitude of these exponents, in turn, quantifies the degree of instability and unpredictability inherent to the system.

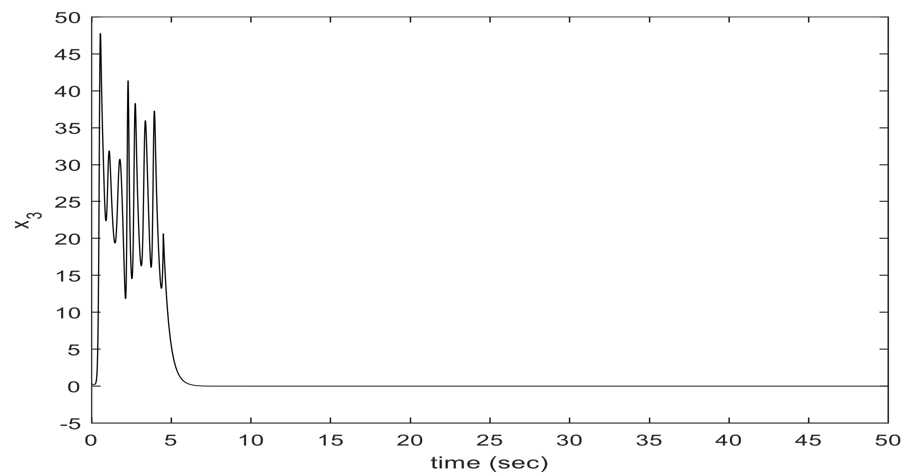
Figures $3(a-f)$ below show the time responses of states with the proposed control functions. The controller of the hyperchaotic system (3) is activated at $t = 20\ secs$. As expected, it shows that the hyperchaotic system can be stabilized to the origin point (0,0,0,0,0). The successful stabilization of the 5D Lorenz system to the origin underscores the controller's capability to tame hyperchaotic behaviour, with far reaching implications for potential applications including complex system control and predictability enhancement.



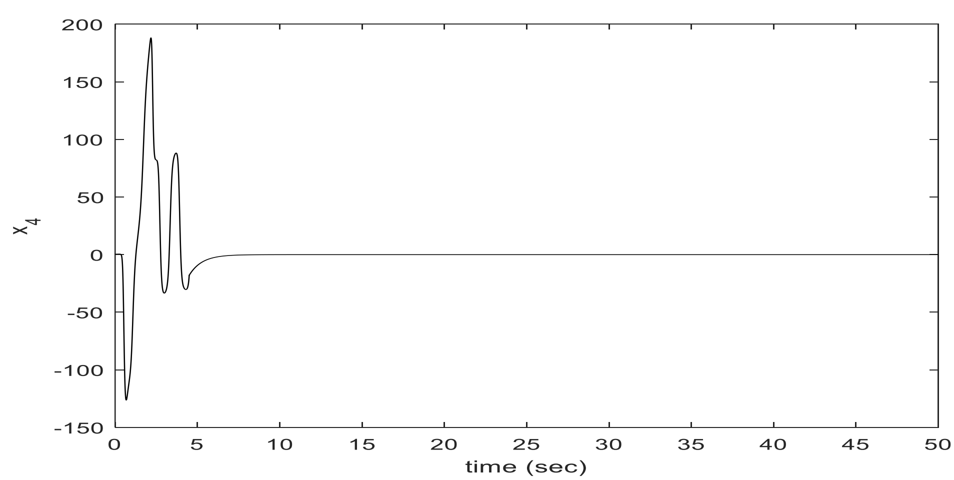
(a) The time response of states x_1



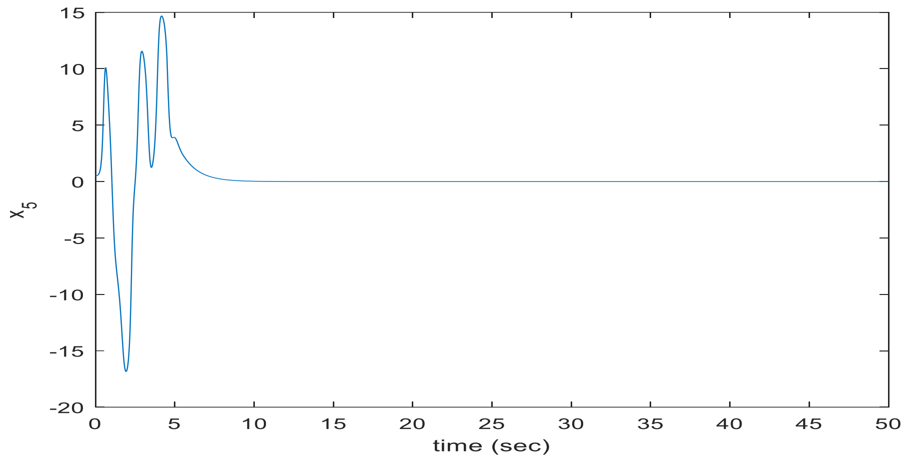
(b) The time response of states x_2



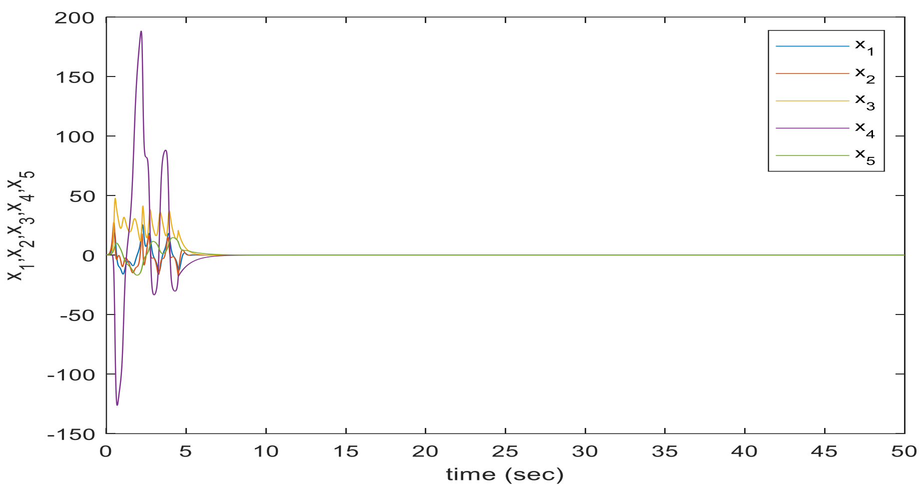
(c) The time response of states x_3



(d) The time response of states x_4



(e) The time response of states x_5

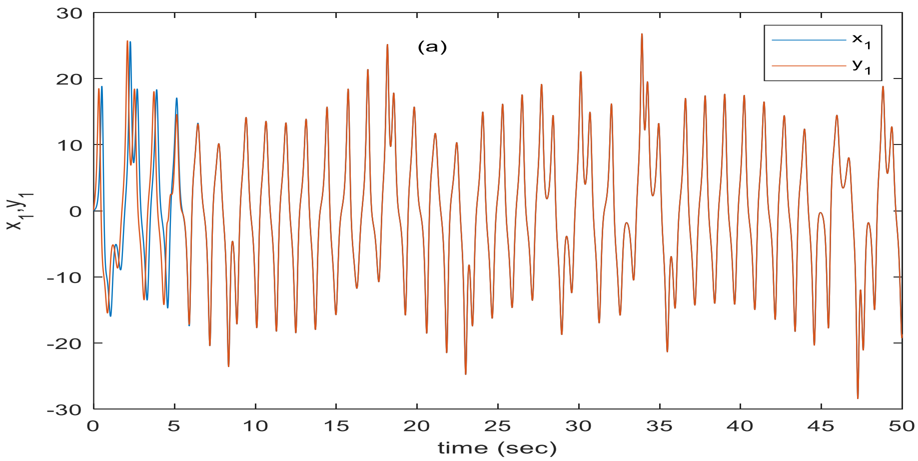


(f) The time response of the controlled states

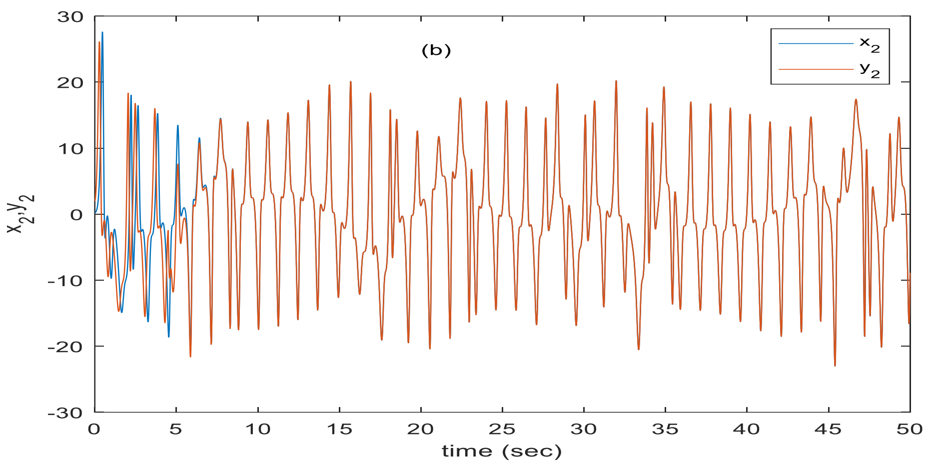
Figure 3(a- f): The time response of states for the hyperchaotic 5D Lorenz system with the proposed controller.

In figure 4(a – e) below, the observed coincidence between the response and drive hyperchaotic Lorenz systems, as revealed by the time series data can be attributed to synchronization. The synchronized state enables control of the response system and allows for predictability of the response system's behaviour based on the behaviour of the drive system. Therefore, the suggested control functions are shown in Figures $4(a-e)$ together with the time responses of states determined by both the drive system and the response system. Figure 5 below displays the synchronization error paths for the drive system and the response system. When $t = 4.5\ secs$, the controller switches on. A synchronization of variables occurs, and it converges to zero. It confirms that all state variables are synchronized, as predicted, and that the synchronization errors are zero. The synchronization of systems (18) and (20) is guaranteed by Figure 6's depiction of the average error state variables' convergence to zero. The synchronization quality E_{av} reported by [53] attests to this.

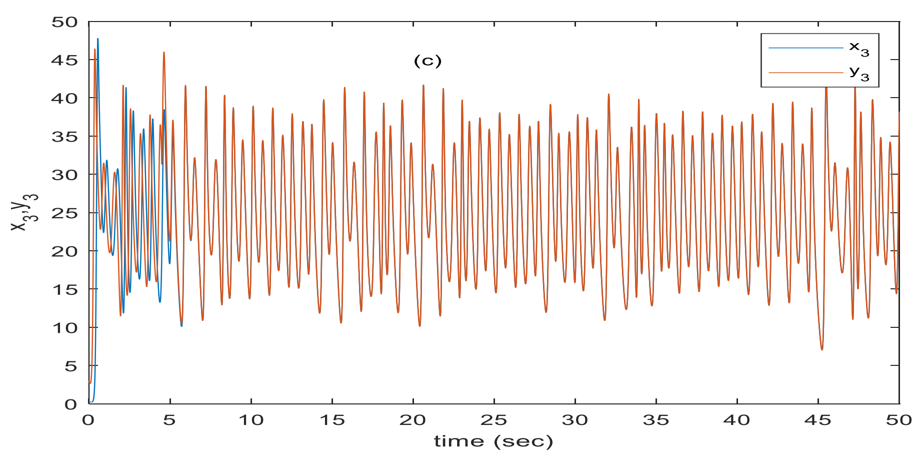
$$E_{av} = \sqrt{e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2} \tag{36}$$



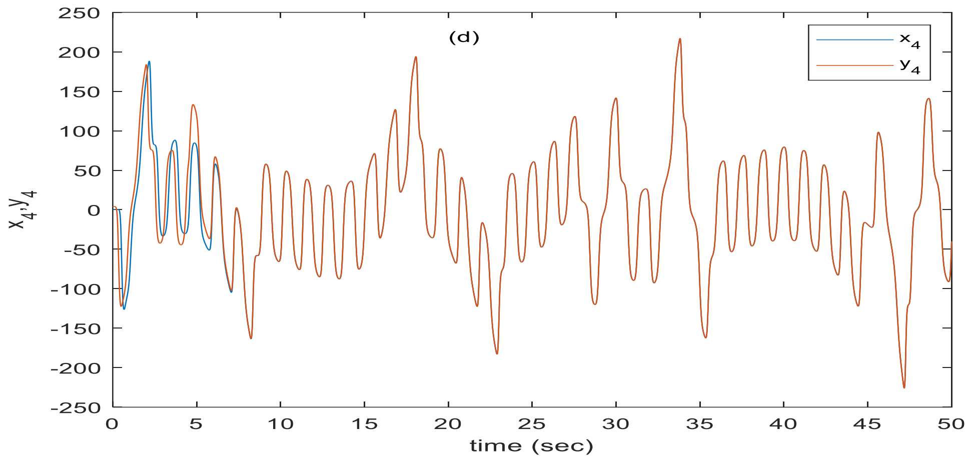
(a) The time response of states x_1 and y_1 of the 5D hyperchaotic Lorenz system



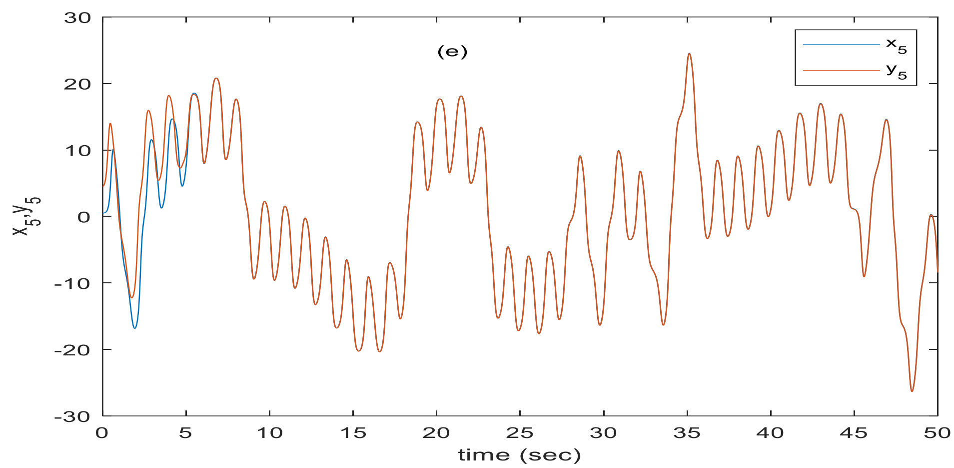
(b) The time response of states x_2 and y_2 of the 5D hyperchaotic Lorenz system



(c) The time response of states x_3 and y_3 of the 5D hyperchaotic Lorenz system



(d) The time response of states x_4 and y_4 of the 5D hyperchaotic Lorenz system



(e) The time response of states x_5 and y_5 of the 5D hyperchaotic Lorenz system.

Figure 4(a – e): The time series data of the hyperchaotic 5D Lorenz systems with the proposed controller.

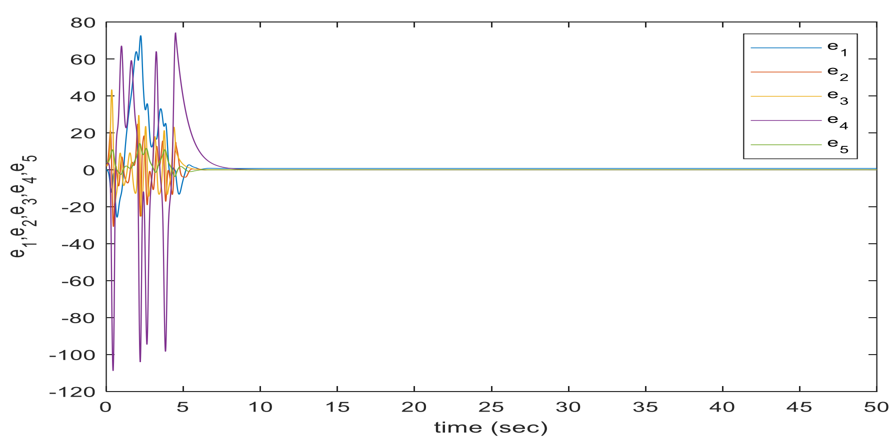


Figure 5: Time response of the global synchronization error of the coupled hyperchaotic 5D Lorenz systems

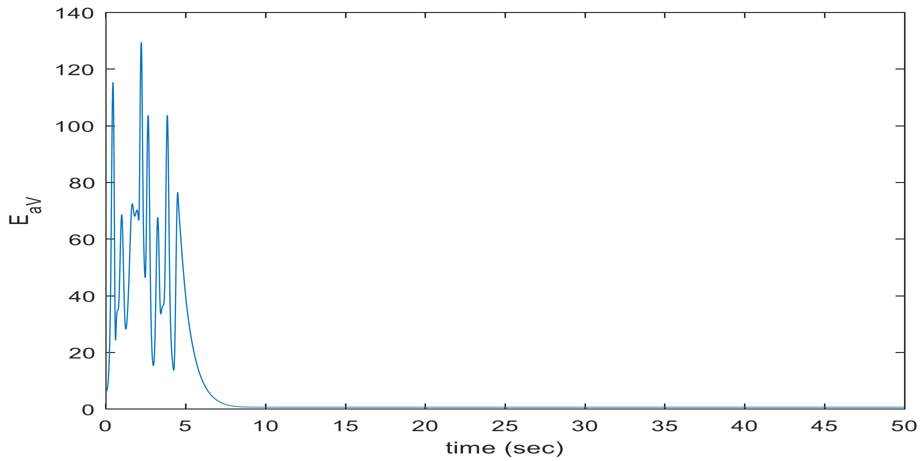


Figure 6: The time response of synchronization average error state for the coupled hyperchaotic 5D Lorenz systems

5 CONCLUSION

In this paper, the active backstepping control approach is used to examine the control and synchronization of the hyperchaotic 5D Lorenz system. The proposed scheme, which combines the selection of a Lyapunov function with the creation of an active control, is a systematic design technique. The proposed control approach is able to stabilize the chaotic motion to the origin. In addition, it systematically synchronizes the two identical hyperchaotic 5D Lorenz systems. It is shown that the 5D hyperchaotic system exhibits three positive Lyapunov exponents and possesses complex dynamical behaviour. Numerical simulations were also carried out to illustrate the effectiveness of the approach. The proposed method is verified to have the following advantages. Firstly, it does not need to calculate the eigenvalues of the Jaccobian matrix. Hence, it is simple and convenient. Secondly, it is applicable to control high dimensional hyperchaotic systems by adopting the active control technique, which has the flexibility to design a control law. Therefore, the dynamics of hyperchaotic systems are better than that of conventional chaotic systems in a variety of applications.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Chen T, Yang H, Yuan J. Event-Triggered Adaptive Neural Network Backstepping Sliding Mode Control for Fractional Order Chaotic Systems Synchronization With Input Delay. Multidisciplinary Rapid Review Open Access Journal. 2021; 9: 14.
2. Teh JS, Tan K, Alawida M. A Chaos-Based Keyed Hash Function Based on Fixed Point Representation. Cluster Computing. 2019; 22(2): 649-60.
3. Wang X, Kuznetsov NV, Chen G. Chaotic systems with multistability and hidden attractors: Springer; 2021.
4. Koronovskii AA, Moskalenko OI, Hramov AE. On the use of chaotic synchronization for secure communication. Physics-Uspekhi. 2009; 52(12): 1213.
5. Tang Y, Li Q, Dong W, Hu M, Zeng R. Optical chaotic communication using correlation demodulation between two synchronized chaos lasers. Journal of Optics Communications 2021; 498(12732).

6. Wu Z, Zhang X, Zhong X. Generalized chaos synchronization circuit simulation and asymmetric image encryption. IEEE Access. 2019; 7: 37989-8008.
7. Kassim S, Megherbi O, Hamiche H, Djennoune S, Bettayeb M, editors. Speech encryption based on the synchronization of fractional-order chaotic maps. 2019 IEEE International Symposium on Signal Processing and Information Technology (ISSPIT); 2019: IEEE.
8. Ouannas A, Batiha IM, Bekiros S, Liu J, Jahanshahi H, Aly AA, et al. Synchronization of the glycolysis reaction–diffusion model via linear control law. Entropy 2021; 23(1516.): 11.
9. Liu LC, Tian B, Xue YS, Wang M, Liu WJ. Analytic solution for a nonlinear chemistry system of ordinary differential equations. Journal of Nonlinear Dyn 2012; 68: 17–21.
10. Li X, Rao R, Zhong S, Yang X, Li H, Zhang Y. Impulsive Control and Synchronization for Fractional-Order Hyper-Chaotic Financial System. Journal of Mathematics. 2022; 10(2737): 13.
11. Tai W, Teng Q, Zhou Y, Zhou J, Wang Z. Chaos Synchronization of Stochastic Reaction-Diffusion Time-Delay Neural Networks Via Non-Fragile Output-Feedback Control. Applied Mathematics and Computation. 2019; 354: 115–27
12. Wang M-H, Lu S-D, Hsieh M-J. Application of extension neural network algorithm and chaos synchronization detection method to partial discharge diagnosis of power capacitor. Journal of Measurement. 2018; 129: 227–35
13. Ayubi P, Setayeshi S, Rahmani AM. Deterministic chaos game: a new fractal based pseudo-random number generator and its cryptographic application. Journal of Information Security and Applications. 2020; 52(102472).
14. Teh JS, Alawida M, Sii YC. Implementation and practical problems of chaos-based cryptography revisited Journal of Information Security and Applications 2020; 50(102421).
15. Xiang S, Han Y, Wang H, Wen A, Hao Y. Zero-lag chaos synchronization properties in a hierarchical tree-type network consisting of mutually coupled semiconductor lasers. . Journal of Nonlinear Dynamics 2020; 99(4): 2893–906.
16. Mohammadi H, Kheshti M. Long-life control of tumor growth via synchronizing to a less severe case Biomedical Signal Processing and Control. 2021; 68(102727).
17. Han X, Cheng F, Tang S, Zhang Y, Fu Y, Cheng W, et al. Synchronization analysis of fractional-order neural networks with adaptive intermittent-active control. IEEE Access. 2022;10:75097-104.
18. Su H, Luo R, Huang M, Fu J. Practical fixed time active control scheme for synchronization of a class of chaotic neural systems with external disturbances. Chaos, Solitons & Fractals. 2022; 157: 111917.
19. Khan A, Chaudhary H, editors. A comprehensive analysis on controlling and hybrid synchronization in identical chaotic systems via active control method. Journal of Physics: Conference Series; 2022: IOP Publishing.
20. Delavari H, Mohadeszadeh M. Hybrid complex projective synchronization of complex chaotic systems using active control technique with nonlinearity in the control input. Journal of Control Engineering and Applied Informatics. 2018; 20(1): 67-74.
21. Shukla M, Sharma B, editors. Hybrid projective synchronization of fractional order Volta's system via active control. 2015 2nd International Conference on Recent Advances in Engineering & Computational Sciences (RAECS); 2015: IEEE.
22. Vaseghi B, Pourmina MA, Mobayen S. Secure communication in wireless sensor networks based on chaos synchronization using adaptive sliding mode control. J Nonlinear Dyn. 2017; 89(3): 1689–704.
23. Alsubaie H, Yousefpour A, Alotaibi A, Alotaibi ND, Jahanshahi H. Stabilization of Nonlinear Vibration of a Fractional-Order Arch MEMS Resonator Using a New Disturbance-Observer-Based Finite-Time Sliding Mode Control. Mathematics. 2023; 11(4): 978.
24. Ramakrishnan B, Tamba VK, Metsebo J, Tokoue Ngatcha D, Rajagopal K. Control, synchronisation and antisynchronisation of chaos in two non-identical Josephson junction models via sliding mode control and its FPGA implementation. Pramana. 2023; 97(1): 46.
25. Chien T-H, Chen Y-C. Combination of observer/Kalman filter identification and digital redesign of observer-based tracker for stochastic chaotic systems. Int Symp Comput, Consum Control (ISC). 2016: 103–7.
26. Handa H. Chaos Synchronization for a Class of Hyperchaotic Systems Using Active SMC and PI SMC: A Comparative Analysis. Journal of Control, Automation and Electrical Systems. 2022; 33(6): 1671-87.

27. Behih K, Saadi S, Bouchama Z. Hyperchaos synchronization using TS fuzzy model based synergetic control theory. International Journal of Intelligent Engineering and Systems. 2021; 14(6): 588-595.
28. Zhang H, Meng D, Wang J, Lu G. Synchronisation of uncertain chaotic systems via fuzzy-regulated adaptive optimal control approach. Int J Syst Sci. 2020; 51(3): 473–487.
29. Solís-Pérez J, Betancourt-Vera J, Gómez-Aguilar J, Alderremy A, Aly S. Non-integer order chaotic systems: numerical analysis and their synchronization scheme via M-backstepping technique. The European Physical Journal Special Topics. 2022; 231(10): 1931-1968.
30. Wang H, Ai Y. Adaptive fixed-time control and synchronization for hyperchaotic Lü systems. Applied Mathematics and Computation. 2022; 433: 127388.
31. Hao Z, Xi-Kui M, Yu Y, Cui-Dong X. Generalized synchronization of hyperchaos and chaos using active backstepping design. Chinese Physics. 2005; 14(1): 86.
32. Li H, Hu Y. Robust sliding-mode backstepping design for synchronization control of cross-strict feedback hyperchaotic systems with unmatched uncertainties. Communications in Nonlinear Science and Numerical Simulation. 2011; 16(10): 3904-3913.
33. Vaidyanathan S, Tlelo-Cuautle E, Sambas A, Dolvis LG, Guillén-Fernández O, Idowu BA. A new multistable hyperjerk dynamical system with self-excited chaotic attractor, its complete synchronisation via backstepping control, circuit simulation and FPGA implementation. International Journal of Modelling, Identification and Control. 2020; 35(3): 177-190.
34. Sabaghian A, Balochian S, Yaghoobi M. Synchronisation of 6D hyper-chaotic system with unknown parameters in the presence of disturbance and parametric uncertainty with unknown bounds. Connection Science. 2020; 32(4): 362-383.
35. Vaidyanathan S, Sambas A, Azar AT, Rana K, Kumar V. A new 5-D hyperchaotic four-wing system with multistability and hidden attractor, its backstepping control, and circuit simulation. Backstepping control of nonlinear dynamical systems: Elsevier; 2021. p. 115-138.
36. Liu B, Sun Z, Luo Y, Zhong Y. Uniform synchronization for chaotic dynamical systems via event-triggered impulsive control. J Phys A, Stat Mech Appl. 2019; 531.
37. Lu X. A financial chaotic system control method based on intermittent controller. Math Problems Eng. 2020; 2020: 1–12.
38. Wang Y, Yu H. Fuzzy synchronization of chaotic systems via intermittent control. Chaos, Solitons & Fractals. 2018; 106: 154-160.
39. He X, Zhang H. Exponential synchronization of complex networks via feedback control and periodically intermittent noise. Journal of the Franklin Institute. 2022; 359(8): 3614-3630.
40. He J, Qiu W, Cai J. Synchronization of Hyperchaotic Systems Based on Intermittent Control and its Application in Secure Communication. Journal of Advanced Computational Intelligence and Intelligent Informatics. 2023; 27(2) :292-303.
41. Wu W, He L, Zhou J, Xuan Z, Arik S. Disturbance-term-based switching event-triggered synchronization control of chaotic Lurie systems subject to a joint performance guarantee. Communications in Nonlinear Science and Numerical Simulation. 2022; 115: 106774.
42. Liu L, Lei M, Bao H. Event-triggered quantized quasisynchronization of uncertain quaternion-valued chaotic neural networks with time-varying delay for image encryption. IEEE Transactions on Cybernetics. 2022.
43. Gu Z, Yan S, Park JH, Xie X. Event-triggered synchronization of chaotic Lur'e systems via memory-based triggering approach. IEEE Transactions on Circuits and Systems II: Express Briefs. 2021; 69(3): 1427-1431.
44. Singh S, Azar AT. Multi-switching combination synchronization of fractional order chaotic system. In Proc Joint Eur-US Workshop Appl Invariance Comput Vis Cham, Switzerland: Springer. 2020: 655–664.
45. Mobayen S, Ma J. Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay. Journal of Chaos, Solitons and Fractals 2018; 114: pp. 46–54.
46. Vaseghi B, Hashemi SS, Mobayen S, Fekih A. Finite Time Chaos Synchronization in Time-Delay Channel and Its Application to Satellite Image Encryption in Orthogonal Frequency Division Multiplexing Communication Systems. Multidisciplinary Rapid Review Open Access Journal. 2021; 9: 13.

47. Shuklaa VK, Mishrac APK. Finite-time synchronization between finance hyper-chaotic systems with hyperbolic nonlinearity via adaptive control. Journal of Scientific Research. 2022; 66(4).
48. Khelifa MA, Boukabou A. Design ofan intelligent prediction-based neural network controller for multi-scroll chaotic system. International Journal of Speech Technology. 2016; 45(3): 793–807.
49. Mihankhah A, Doustmohammadi A. Adaptive neural-based finite-time attitude synchronization and tracking control of multiple rigid bodies under actuator faults and saturation. Aircraft Engineering and Aerospace Technology. 2021; 94(3): 407-417.
50. Li K, Cao J, He J-M. Hidden Hyperchaotic Attractors in a New 4D Fractional Order System and Its Synchronization. Journal of Chaos. 2020; 30.
51. Jahanshahi H, Yousefpour A, Wei Z, Alcaraz R, Bekiros S. A Financial Hyperchaotic System With Coexisting Attractors: Dynamic Investigation, Entropy Analysis, Control And Synchronization. Nonlinear Analysis: Modelling and Control. 2015.
52. Wang B, Dong X. Secure Communication Based on a Hyperchaotic System with Disturbances. Math Problems Eng. 2014; 2015.
53. Vincent U.E, *Chaos Synchronization Using Active Control and Backstepping Control: A Comparative Analysis* Nonlinear Analysis: Modelling and Control, 2008. 13(2): p. 253–26